



MJ-3704

First Year B. C. A. (Sem. I) Examination

December – 2015

Paper : 102 - Mathematics

Time : Hours]

[Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fillup strictly the details of signs on your answer book. Name of the Examination : <b>First Year B. C. A. (Sem. I)</b> Name of the Subject : <b>Paper : 102 - Mathematics</b> Subject Code No. : <b>3 7 0 4</b> Section No. (1, 2,.....) : <b>Nil</b>	Seat No. : <table border="1"> <tr> <td> </td><td> </td><td> </td><td> </td><td> </td><td> </td> </tr> </table>						
	Student's Signature						

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

Q:1 Answer the following Questions:

[10]

1. Define Equivalent set with illustration:
2. When do you say that a function is one-to-one?
3. Define skew symmetric matrix with illustration.
4. Define logical connectives, give examples of at least two.
5. Define value of Boolean Expression with example.

Q: 2 (A) Verify Distributive law of union of  $A = \{x \leq 5; x \in N\}$  over intersection of  $B = \{x : x^2 \leq 9; x \in Z\}$

and  $C = \{x : -1 \leq x \leq 4; x \in N\}$

[05]

OR

Q: 2 (A) in usual notations prove that  $A \times (B \cup C) = (A \times B) \cup (A \times C)$ .

[05]

Q: 2 (B) Attempt any two:

[10]

(1) If  $A = \{a / a^2 - 1 < 10; a \in N\}$ ,  $B = \{b / b - 1 < 2; b \in N\}$  and  $C = \{c / |c| \leq 1; c \in Z\}$  then verify that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ .

(2) If  $U = \{x : x \leq 10; x \in N\}$ ,  $A = \{x : x^2 < 10; x \in N\}$ ,  $B = \{2, 4, 6\}$  &  $C = \{x : x^3 - 3x^2 - 4 = 0; x \in R\}$

Then verify that (i)  $A \cap (B - C) = (A \cap B) - (A \cap C)$

(ii)  $A' - B' = B - A$

(3) Prove that :

(i)  $A - (A - B) = A \cap B$

(ii)  $(A \cap B)' = A' \cup B'$

(4) A town has a total population of 50,000 person and of them 28000 read 'Gujarat Samachar' and 23000 read 'Sandesh' while 4000 read both the papers .prove that there are 3000 persons who read neither of the both.

Q: 3 (A) If  $f(x) = \frac{1}{x}, x \in Z - \{-1, 0, 1\}$  then prove that  $f(x+1) - f(x-1) = \frac{2}{1-x^2}$  [05]

OR

Q: 3 (A) It is observed that a quadratic function  $y = ax^2 + bx + c$  fits the points (1,8), (1,4) and (2,5) find the constants  $a, b, c$  and rewrite the function, also find value of  $y$  when  $x = 4$ . [05]

Q: 3 (B) Attempt any two: [10]

1. If  $f(x) = x^3 - 2x + \frac{1}{x}$  then find  $f(x) + f(-x)$ .

2. If  $f(x) = x^2 + 4x + 5$  and  $g(x) = 2x + 1$  then prove that  $f(1) - 2g(2) = 0$ .

3. The fixed cost of Transistors is Rs. 2,00,000 and the variable cost is Rs. 1000 per unit. if the selling Price is Rs. 1500, then find

(i) Cost function (ii) Revenue function, (iii) Breakeven point

4. If  $f(x) = 2x^2 - 1$  and  $g(x) = 2x - 1, x \in \{0, 1, 2\}$ , are the functions equal?

Q:4 (A) Show that  $D_{12}$  is a Boolean Algebra where  $\forall a, b \in D_{12}$  [05]

$$a + b = \text{L.C.M. of } a, b$$

$$a \cdot b = \text{G.C.D. of } a, b$$

$$a' = 12 / a$$

OR

Q:4 (A) Prove that the argument in the following example is not logically valid [05]

Hypothesis:  $\begin{cases} S_1 : p \wedge (\neg q) \Rightarrow r \\ S_2 : p \vee q \\ S_3 : q \Rightarrow p \end{cases}$  Conclusion:  $S : r$

Q: 4(B) Attempt Any two [10]

1 Using Truth table prove that

$$(i) (p \Rightarrow q) = [(p \wedge \neg q) \Rightarrow (\neg p)] \quad (ii) \neg(p \Rightarrow q) = p \wedge (\neg q)$$

2. Construct the input/output table for

$$(i) f(x) = (x_1, x_2, x_3) = (x_1 \cdot x_2)' + x_3 \quad (ii) f(x) = (x_1, x_2) = (x_1 \cdot x_2) + x_2$$

3. In a Boolean Algebra  $B$ , prove that  $x + 1 = 1$  and  $x \cdot 0 = 0; \forall x \in B$

4. Find the product sum canonical form of  $f(x_1, x_2) = x_1 \cdot x_2 + x_1' \cdot x_2 + x_1 \cdot x_2'$

Q:5(A) If  $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & -5 \\ 2 & 0 & 4 \end{bmatrix}$  then find  $A^{-1}$  also verify that  $A^{-1} \cdot A = I$  [05]

OR

Q: 5(A) If  $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$  then

(i)  $A \cdot B = B \cdot A = 0$

Prove that (ii)  $A \cdot C = A$

(iii)  $C \cdot A = C$

[05]

Q: 5(B) Attempt Any TWO

[10]

1. If  $A^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 6 & -3 & 0 \end{bmatrix}$ ,  $B^{-1} = \begin{bmatrix} 6 & 3 & 1 \\ 2 & 4 & -8 \\ 3 & -6 & 1 \end{bmatrix}$  then find  $(B \cdot A)^{-1}$

2. Solve the following system of equations using crammer's rule

$$x + 6y = 2xy$$

$$3x + 2y = 2xy$$

3. Show that  $D_{21}$  is a Boolean Algebra where  $\forall a, b \in D_{21}$

$$a + b = \text{L.C.M. of } a, b$$

$$a \cdot b = \text{G.C.D. of } a, b$$

$$a' = 21/a$$

4. . Solve the following system of equations using crammer's rule

$$x + 2y = 3$$

$$y - 3z = 4$$

$$3x - 2z = 5$$