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H-3503

B. C. A. (Sem. I) Examination

March / April - 2018

102 : Mathematics - I

Time : 3 Hours] [Total Marks : 70

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ B. C. A. (SEM. 1)

Name of the Subject :
☛ 102 : MATHEMATICS - I

☛ Subject Code No. : 3 5 0 3 ☛ Section No. (1, 2,.....) : Nil

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate marks of corresponding question.

(4) Follow usual notations.

(5) Use of non-programmable scientific calculator is allowed.

1 Answer the following : **10**

(1) Define : Power set.

(2) If $A=\{1,2,3,4\}, B=\{1,2,5,6\}$ then find $A-B$

(3) If $f(x)=x^2-x+1$ then find $f(0)+f(-1)$

(4) Define : Many-one function.

(5) Define : Conjunction.

(6) Define : Tautology.

(7) In a Boolean Algebra prove that $0'=1$ and $1'=0$.

(8) Define : Principle of duality.

(9) Evaluate : $\begin{vmatrix} -6 & 2 \\ -3 & -4 \end{vmatrix}$

(10) If $A=\begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$ then find adj. A.

2 (A) In usual notations prove that 5

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

OR

(A) In usual notations prove that

$A \times (B \cup C) = (A \times B) \cup (A \times C)$

(B) Attempt any **two** : 10

(1) In a college there are 500 students and out of them 300 have taken Economics and 250 have taken Statistics. All students have taken at least one of these two subjects. How many of them have taken both the subjects?

(2) If $A = \{1,3\}$, $B = \{3,5\}$ and $C = \{3,5,6\}$ then verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(3) If $A = \{1,3,4,6\}$, $B = \{2,4,5\}$ and $C = \{3,4,5\}$ then verify that $A \cap (B - C) = (A \cap B) - (A \cap C)$

(4) If $A = \{x \mid x \in N; 2 < x < 6\}$, $B = \{x \mid x \in N; x^2 < 5x\}$ and $U = \{x \mid x \in N; x < 10\}$ then prove that $(A \cup B)' = A' \cap B'$

3 (A) If $f(x) = \frac{1}{x} + \frac{2}{x-3}; x \in R - \{0,3\}$ then find $f(1), f(2), f(-3), f\left(\frac{1}{3}\right)$ 5

OR

(A) If $f(x) = x(x+1)(2x+1)$ then prove that $f(x) - f(x+1) = 6x^2$.

(B) Attempt any **two** : 10

(1) If $f(x) = x^3$ and $g(x) = 3x^2 - 2x$ where $D_f = D_g = \{0,1,2\}$ is $f = g$? Justify your answer.

(2) It is observed that a quadratic function $ax^2 + bx + c$ fits the data points (1, 9), (2,14) and (3, 23). Find the constants a, b and c and find y when $x = 4$.

(3) If $f(x)=\dfrac{x^2-x}{x+3}$ then find $\dfrac{f(0)+f(-2)}{f(1)+f(3)}$

(4) If $f(x)=x^2+4x+5$ and $g(x)=2x+1$ then prove that $f(1)-2g(2)=0$.

4 (A) $A=\begin{bmatrix}0 & 4 & 3\\ 1 & -3 & -3\\ -1 & 4 & 4\end{bmatrix}$ then prove that $A^2=I$. 5

OR

(A) If $A=\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$ then prove that $A^2-4A-5I=0$.

(B) Attempt any **two** : 10

(1) If $A=\begin{bmatrix}2 & -1 & 3\\ 3 & 6 & 8\\ 5 & 0 & 9\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4 & 9\\ -3 & 0 & 2\\ 7 & 6 & 5\end{bmatrix}$ then show that $(A+B)^T=A^T+B^T$.

(2) Solve the following equations by Cramer's Rule : $x+2y+3z-14=0$; $2x+y+z-7=0$; $5x+2y+z-12=0$.

(3) Solve the following equations by Cramer's Rule : $x+6y=2xy$, $3x+2y=2xy$.

(4) Find inverse of the matrix $A=\begin{bmatrix}1 & 2 & 3\\ 2 & 4 & -8\\ 6 & -3 & 0\end{bmatrix}$

5 (A) Let $B=\{0,1\}$. Prepare an input/output table for the boolean function $f:B^2\to B,\ f(x_1,x_2)=x_1\cdot x_2'$ 5

OR

(A) Check the validity of the following argument :

Hypothesis $S_1:p\Rightarrow(\sim q),\ S_2:r\Rightarrow q,\ S_3:r$

Conclusion : $S:\sim p$

(B) Attempt any two : **10**

(1) Show that $\{p \Rightarrow (q \Rightarrow r)\} \Rightarrow \{(p \Rightarrow q) \Rightarrow (p \Rightarrow r)\}$ is a tautology.

(2) Using truth tables prove that $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$.

(3) Express boolean function $f(a,b,c) = (a \cdot b) + (a \cdot c) + (b \cdot c)$ as a product of sums in three variables.

(4) For the element x, y of a boolean algebra, prove that $x \cdot y' = 0 \Leftrightarrow x \cdot y = x$
