RA-3704

First Year B. C. A. (Sem. I) Examination

March / April - 2017

Mathematics : Paper - 102

Time : **3** Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :
☛ **FIRST YEAR B. C. A. (SEM. 1)**

Name of the Subject :
☛ **MATHEMATICS : PAPER - 102**

☛ Subject Code No. : **3 7 0 4** ☛ Section No. (1, 2,.....) : **Nil**

Seat No. :

Student's Signature

(2) All questions are compulsory.

(3) Figures to the right indicate full marks.

1 Answer the following questions : **10**

(1) Define equivalent sets with illustration.

(2) Explain symmetric difference of two non-empty sets with illustration.

(3) Define complement of set with illustration.

(4) Define onto function with illustration.

(5) Find $f\left(\frac{2}{3}\right) - f\left(\frac{3}{2}\right)$ for $f(x) = x^2 + x - 1$.

(6) Define Domain of the function and find D_f for $f(x) = 2_x - 3\ R_f = \{-3,1,0\}$.

(7) Solve the equation $\begin{vmatrix} -(x-y) & -x \\ x & (x+y) \end{vmatrix} = 2$

(8) Define Transpose of matrix with illustration.

(9) Define Boolean function.

(10) Define Universal quantifier and Existential quantifier.

2 (A) In usual notations prove that $A-(B\cup C)=(A-B)\cap(A-C)$ **5**

OR

(A) In usual notations prove that $A\times(B\cup C)=(A\times B)\cup(A\times C)$ **5**

(B) Attempt any **two** : **10**

(1) Let $U=\{x/2<x<14, x\in N\}$, $A=\{z/3\le z\le 8, z\in N\}$ and $B=\{y=2n+1, y\le 12, n\in N\cup\{0\}\}$ then find $(i) A', (ii) B' (iii) (A\cup B)'$

(2) If $A=\{x\le 3; x\in N\}, B=\{x:-1\le x\le 2; x\in Z\}$ and $C=\{x:x^2-5x+6; x\in R\}$ considering $U=R$, verify DeMorgan's law for intersection.

(3) If $A=\{x\le 4; x\in N\}, B=\{x:x^2\le 4; x\in Z\}$, and $C=\{x:-2\le x\le 3; x\in N\}$ then verify that $A-(B\cap C)=(A-B)\cup(A-C)$.

(4) In a class of 42 students, each play at least one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey.

3 If $f(x)=x^2(x-1)^2, x\in R$ then prove that $f(x+1)-f(x)=4x^3$ **5**

OR

3 (A) The demand function of a commodity is $d=f(p)=1605-5p^2$, find demand when price is Rs. 5,6 and 8 respectively. At what price the demand will be zero? **5**

(B) Attempt any two : **10**

(1) If $f(x)=\frac{1}{x}+\frac{2}{x-3}; x\in R-\{0,3\}$ then find $f(1/3)-f(-3)+f(2)$.

(2) Fixed cost of a factory producing particular types of bag is Rs. 9000 and the variable cost per bag is Rs. 110. If the selling price per bag is Rs. 240 then find profit function.

(3) If $f(x)=\dfrac{ax+b}{cx-a}$ then prove that $x=f(y)$

(4) If $f(x)=x^3$ and $g(x)=3x^2-2x, x\in\{0,1,2\}$ are the functions equal ?

4 (A) Show that D_{18} is a Boolean Algebra where $\forall a,b\in D_{18}$. **5**

$a+b=$ L.C.M. of a,b

$a\bullet b=$ G.C.D. of a,b

$a'=18/a$

OR

(A) Prove that the argument in the following example is not logically valid **5**

$$\text{Hypothesis}:\begin{cases}S_1: p\Rightarrow q\\ S_2: p\Rightarrow r\end{cases}\quad \text{Conclusion}: S: p\Rightarrow(q\wedge r)$$

(B) Attempt any **two** : **10**

(1) Prove that the following propositions are tautologies

(i) $[(\sim q)\Rightarrow(\sim p)]\Rightarrow(p\Rightarrow q)$

(ii) $\sim(p\Rightarrow q)\vee(\sim p\wedge q)\vee p$

(2) Using Truth table show that

(i) $P\Rightarrow(q\vee r)\cong(p\Rightarrow q)\vee(p\Rightarrow r)$

(ii) $(p\vee q)\Rightarrow r\cong(p\Rightarrow r)\wedge(p\Rightarrow r)$

(3) In a Boolean algebra the complement of an element is unique.

(4) In a Boolean algebra B prove that $\left.\begin{array}{l}(1)\ x.0=0\\(2)\ x+1=1\end{array}\right\};\forall x\in B.$

5 (a) If $A = [2 \ 4 \ 6]$, $B = \begin{bmatrix} 2 & 3 & 0 \\ 0 & 4 & 9 \\ 9 & 1 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$ then find $A \cdot B \cdot C$. **5**

OR

(a) Let $A = \begin{bmatrix} 1 & 0 & 7 \\ 2 & 2 & 5 \\ 0 & 3 & 6 \end{bmatrix}$, is A singular ? find A^{-1} **5**

(b) Attempt any **two** : **10**

(1) If $A = \begin{bmatrix} -1/3 & 3/5 & 2/7 \\ 4/5 & -7/9 & 1/2 \\ 3/4 & 1/7 & -2/5 \end{bmatrix}$, $B = -A$ and $C = -2B$ then find $2A + B - C$.

(2) Solve the following system of equations using Crammer's rule

$ax + by = ab$

$bx + ay = ab$

(3) Show that D_{12} is a Boolean Algebra where $\forall a, b \in D_{12}$

$a + b =$ L.C.M. of a, b

$a \cdot b =$ G.C.D. of a, b

$a' = 12/a$

(4) If $A = \begin{bmatrix} 3 & 1 & 1 \\ -2 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$ then find $A^2 + 2A - 3I$.